It is necessary for the transformer designers to know the electrical stresses at all parts of the windings as accurately as possible at the design state itself so that they may select an efficient winding and insulation structure.

In this paper a proper winding model has been selected for the stress analysis. The trapezoidal rule of integration is used for the computation of impulse voltage stresses in transformer winding through a companion network approach. Because the trapezoidal rule of integration method is a numerically stable with reasonable accuracy, it is proposed to employ this method for calculating the transient phenomena in transformer winding. The transient-voltage response is calculated using trapezoidal rule of integration through a companion network approach. The simulation is done using EMTP.

For the purpose of certification as per IS2026 impulse test has to be conducted on power transformers. The main components of tests are a standard 1/50 impulse voltage of prescribed magnitude followed by chopped voltage test, which is again followed by 1/50 impulse voltage test. When the chopped voltage test is applied, the transformer winding will be severely stressed. So if the transformer winding fails at any location this will be shown in oscillogram taken. Usually for this purpose oscillogram of the neutral currents of the full impulse voltage applied before and after the chopped voltage is compared. If there is no change in the two oscillogram, the transformer is deemed to have passed the impulse voltage test. If there is a change that means it is due to the inter turn insulation failure or major insulation failure. However from the oscillogram one can not find out where the failure has occurred. Digitally simulating the various inter turn insulation failures by shorting the corresponding two nodes current records can be obtained and this can be compared with the oscillogram obtained in the actual impulse voltage test which will help to find out the actual location of the inter turn insulation failure.

We have simulated the partial discharge across every node and node, node and ground in transformer winding. So that in practice if we found any partial discharge in transformer windings, we can compare it with our templated results and faults can be easily diagnosed.

**STRESS IN TRANSFORMER WINDING:**

It is necessary for the transformer designer to know the electrical stresses, for example impulse, switching, chopped waves stresses at all parts of the winding as accurately as possible at the design state itself so that he may select an efficient winding and insulation structure.

Although very accurate determination of the impulse voltage distribution in transformer winding is possible with the help of the electromagnetic model
developed by “Abetti”, the method lacks flexibility because a new model has to be constructed each time and a new design is to be studied. Further the construction of a model is both time consuming and costly. Since the digital computer came in to use there has been a growing interest in calculating numerically the transient phenomena in transformer windings.

Any numerical method used for the purpose should be stable, require minimum computer time and storage and give results of acceptable accuracy for engineering purposes. Because the trapezoidal rule of integration is a numerically stable method with reasonable accuracy, it was proposed to employ this method for calculating the transient phenomena in transformer windings.

Among all the apparatus subjected to transients in power system, transformers are given the greatest attention because they are much more exposed to surges, especially those caused by lightning.

It is helpful to divide the time after surge strikes the winding into three time intervals. The first of these periods is extremely short, usually a fraction of a millisecond. During this time no significant current can penetrate the winding proper because of its inductance. Currents flow only as displacement currents in the capacitance of the winding. This gives rise to an initial voltage distribution in the winding has reached some steady state. During the intervening or second period the voltage pattern goes through severe contortions as it relaxes from the initial to this final stage. The transients’ calculations in this project are going to be done in this intervening period.

Let us see how the winding is stressed. The oscillations due to the surge writhes in a most tormented way.
We are familiar with the idea of the potential of a point starting a transient at some initial value, oscillating about some final value and in the process swinging almost as far above the steady value as it starts below it.

**PROBLEM FORMULATION:**

The problem is formulated through a companion network approach. The companion network is the original network in which each branch element is replaced by its companion model. The companion model of a branch element is obtained from a network interpretation of the recursion relationship, which results from the application of a numerical integration formula to the branch relationship. The companion models for different branch elements, derived from the application of the trapezoidal rule of integration. In the companion model, the resistor \( R \) remains fixed for fixed step size \( \Delta t \). The current source depends on the voltage \( U(t_k) \) that is determined previously at time \( t_k \). The companion models are also known as the associate discrete circuit models.

**PRACTICAL MODEL:**

A proper mathematical model of the transformer winding is necessary for computing the impulse voltage response by means of a digital computer. For this purpose the traditional equivalent circuit shown in fig1 is used.

---

**fig1.** The circuit model chosen.

The transformer winding is subdivided into a finite number of sections, each containing inductance, series and ground capacitances. The inductive elements are all mutually coupled to each other. The capacitance between nonadjacent sections and losses in the transformer are neglected for simplicity.

**APPROACH:**

For obtaining the companion network the circuit shown in fig1 is replaced by an equivalent circuit with uncoupled elements. The two circuits are equivalent and have the same nodal admittance matrix. The circuit with uncoupled elements can therefore be synthesized from the nodal admittance matrix of the circuit in fig1. If the transformer winding is subdivided into \( N \) sections, there will be \( N \) node pairs. The nodal admittance matrix \( Y_N \) of the circuit in fig1 is therefore a \( N \times N \) matrix. The procedures for obtaining \( Y_N \) will be dealt later.
To construct the equivalent network with uncoupled elements shown in fig2 from the nodal admittance matrix $Y_N$, the following procedure is applied.

fig2. Equivalent circuit without mutual coupling elements

A matrix element $Y_{Nij}$ is represented as an admittance branch $Y_{Nij}$ drawn between the nodes $i$ and $j$. When all the elements of a single element row, except the one on the main diagonal, have been accounted for, the sum of all the off diagonal elements is added to the diagonal term to get the admittance, which must be connected between node $i$ and reference node. The resulting network is equivalent to the original network with respect to the response at the node pairs having a common reference node. This procedure yields a circuit without mutual coupling of branch elements, which is immediately adaptable for writing the companion network. The companion network is obtained by merely replacing each branch element of the circuit in fig2 by its companion model.

The transformer winding is subdivided into a finite number of sections (in this paper I have taken the model contains 10 coils) each containing inductance, series and ground capacitances. The inductive elements are all mutually coupled to each other. The capacitance between nonadjacent sections and the losses in the transformer are neglected for simplicity.

As the impulse surge strikes the winding in a short period, no significant flux can penetrate the iron core. Therefore we are considering the leakage inductance only.

The problem is formulated through a companion network approach. The companion network is the original network in which each branch element is replaced by its companion model.

For obtaining the companion network the circuit shown in figure is replaced by an equivalent circuit which consists of no mutual coupling elements.

To derive this equivalent, first I have taken the circuit which consists of 3 coils. The coils are mutually coupled with each other.
I have formed the nodal admittance matrix from which the equivalent network without mutual coupled elements is to be derived. i.e. for example to find the elements in first row of admittance the following equations are written.

**NODAL EQUATIONS**

- **In first node,**
  - \( E_1 = \omega \left[ i_1 L - i_2 M_{12} - i_3 M_{13} \right] \)
  - \( 0 = \omega \left[ i_2 L - i_1 M_{12} - i_3 M_{23} \right] \)
  - \( 0 = \omega \left[ i_3 L - i_1 M_{13} - i_2 M_{23} \right] \)

\[
\begin{pmatrix}
L & -M_{12} & -M_{13} \\
-M_{12} & L & -M_{23} \\
-M_{13} & -M_{23} & L
\end{pmatrix}
\begin{pmatrix}
i_1 \\
i_2 \\
i_3
\end{pmatrix}
=
\begin{pmatrix}
E_1 \\
0 \\
0
\end{pmatrix}
\]

- By solving the equations \( i_1, i_2, i_3 \) are calculated.
- Now,
  - \( I_1 = i_1 = Y_{11} \)
  - \( I_2 = i_2 - i_1 = Y_{12} \)
  - \( I_3 = i_3 - i_2 = Y_{13} \)
- Like this, in second node nodal equations,
-E2=ω [i1L-i2M12-i3M13]
E2 =ω [i2L-i1M12-i3M23]
0 =ω [i3L-i1M13-i2M23]

By solving the equations \(i_1, i_2, i_3\) are calculated.

Now,
\[
\begin{align*}
I_1 &= i_1 = Y_{21}; \\
I_2 &= i_2 - i_1 = Y_{22}; \\
I_3 &= i_3 - i_2 = Y_{23};
\end{align*}
\]

To form the admittance matrix, I have developed a program in C++.

Thus the elements in nodal admittance matrix are formed.
I have prepared a program to form the nodal admittance matrix in C++ for larger model. After this capacitance is included and the companion network will be achieved.

**ADMITTANCE MATRIX**

\[
\begin{bmatrix}
L & -M_{12} & -M_{13} \\
-M_{12} & L & -M_{23} \\
-M_{13} & -M_{23} & L
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2 \\
i_3
\end{bmatrix}
= \begin{bmatrix}
-E_2 \\
E_2 \\
0
\end{bmatrix}
\]

| 0.513313 | -0.167277 | -0.100461 | -0.141711 | -0.147996 | -0.107748 | -0.044963 | 0.02742 | 0.098094 |
| -0.167277 | 0.464972 | -0.178448 | -0.066555 | -0.069951 | -0.061984 | -0.046637 | -0.022172 | 0.009535 | 0.040422 |
| -0.100461 | -0.178449 | 0.465887 | -0.155954 | -0.026953 | -0.020396 | -0.012615 | -0.007558 | -0.000457 | 0.009535 |
| -0.141711 | -0.066555 | -0.155954 | 0.503545 | -0.10981 | 0.01966 | 0.017256 | 0.008262 | -0.007558 | -0.022172 |
| -0.147996 | -0.069951 | -0.026953 | 0.548513 | -0.10981 | 0.01966 | 0.017256 | 0.008262 | -0.007558 | -0.022172 |
| -0.107748 | -0.06637 | -0.020396 | -0.012615 | 0.548513 | -0.10981 | 0.01966 | 0.017256 | -0.007558 | -0.022172 |
| -0.044962 | -0.022172 | -0.007558 | 0.008262 | 0.017256 | 0.01966 | 0.017256 | -0.10981 | 0.01966 | 0.017256 |
| -0.044963 | -0.046637 | -0.061984 | -0.069515 | -0.022172 | -0.046637 | 0.01966 | 0.017256 | -0.10981 | 0.01966 |
| 0.02742 | 0.009535 | -0.007558 | 0.008262 | 0.017256 | 0.01966 | 0.017256 | -0.10981 | 0.01966 | 0.017256 |
| 0.098094 | 0.040422 | 0.009535 | -0.022172 | -0.046637 | -0.061984 | -0.069515 | -0.022172 | -0.046637 | 0.01966 |
| 0.098094 | 0.040422 | 0.009535 | -0.022172 | -0.046637 | -0.061984 | -0.069515 | -0.022172 | -0.046637 | 0.01966 |
| 0.098094 | 0.040422 | 0.009535 | -0.022172 | -0.046637 | -0.061984 | -0.069515 | -0.022172 | -0.046637 | 0.01966 |

Trace:
0.009535 0.009535 0.009535
The equivalent network without mutual coupling obtained for the 10 coils circuit is as shown below.

![Equivalent circuit without mutual coupling elements](image)

**fig3. Equivalent circuit without mutual coupling elements**

**USAGE OF EMTP:**

From this circuit the transient studies are undertaken. For applying trapezoidal rule of integration and to measure the electromagnetic transients, EMTP package is used and the results are obtained. First the step response is studied and the response for the impulse waveform is studied. The results are included in the appendix.

**FAULT DIAGNOSIS:**

**PARTIAL DISCHARGE SIMULATION IN TRANSFORMER WINDING:**

Solid insulating materials and to a lesser extend liquid dielectrics contain voids or cavities within the medium or at the boundaries between the dielectrics and the electrodes. These voids are generally filled with a medium of lower dielectric strength and the dielectric constant of the medium in the voids is lower than that of the insulation. Therefore, even under normal working voltages the field in the voids may exceed their breakdown value, the breakdown may occur.

When a voltage V is applied across the insulator, the voltage across the void reaches the breakdown strength of the medium in the cavity (Vi) and breakdown occurs. Vi is called the “discharge inception voltage”. When the applied voltage is a.c, breakdown occurs on both the half cycles and the number of discharges will depend on the applied voltage. When the first breakdown across the cavity occurs, the breakdown voltage across it becomes zero. When once the voltage V1 becomes zero, the spark gets extinguished and again the voltage rises till breakdown occurs again. This
process repeats again and again, and current pulses will be obtained both in the positive and negative half cycles.

When the breakdown occurs in the voids, electrons and positive ions are formed. They will have sufficient energy and when they reach the void surfaces they may break the chemical bonds. Also in each discharge there will be some heat dissipated in the cavities, and this will carbonize the surface of the voids and will cause erosion of the material. Channels and pits formed on the cavity surfaces increase the conduction. Chemical degradation may also occur as a result of the active discharge products formed during breakdown. All these effects will result in a gradual erosion of the material and consequent reduction in the thickness of insulation leading to breakdown. The life of the insulation with internal discharges depends upon the applied voltage and the number of discharges. Breakdown by this process may occur in a few days or may take a few years.

An electrical equivalent circuit as shown in fig below can represent electrical insulation with imperfection or voids leading to partial discharges.

Consider a capacitor with a void inside the insulation (Ca). The capacitance of the void is represented by a capacitor in series with the rest of the insulation capacitance (Cb). The remaining void free material is represented by the capacitance (Cc). When the voltage across the capacitor is raised, a critical value is reached across the capacitor Ca and a discharge occurs through the capacitor. ie it becomes short circuited. Generally Ca<<Cb<<Cc. A change $\Delta q_a$, which was present in the capacitor Ca giving, rise to a voltage pulse across the capacitor gives the amount of discharge quality. But this measurement is difficult in practice, and an apparent charge measurement across deleting impedance is usually made.
The circuit arrangement shown in fig gives a simplified circuit for detecting partial discharges. The high voltage transformer shown is free from internal discharges. A resonant filter is used to prevent any pulses starting from the capacitance of the winding and pushing of the transformer. \( C_x \) is the text object, \( C_c \) is the coupling capacitor and \( Z_m \) is the detection impedance. The signal developed across the impedance \( Z_m \) is passed through a band pass filter and amplifier and displayed on a CRO or counted by a pulse counter multi-channel analyzer unit.

The discharge pattern displayed on the CRO screen of a partial discharge detector is shown in fig.

![Discharges in voids in the insulation](image)

This pattern of discharge appears on the quadrants of the ellipse which correspond to the test collapse rising from zero to max, either positively or negatively. The discharges usually start near the peaks of the test voltage but spread towards zero value as the test voltage is increased beyond the inception level. The number and magnitude of the discharges on both the +ve and -ve cycles are approximately the same.

Partial discharge can occur across the inter turn capacitor or the shunt capacitor i.e between node and ground. So it is proposed to simulate the discharge by connecting a current source between each node and ground for simulating the shunt capacitor discharge and a positive current source is connected in one node and a negative current source connecting in the other node to simulate the partial discharge across the inter turn capacitor. In both cases the voltages waveform is obtained in the first node, which is the input terminal of the transformer terminal. These voltages can be calculated at the design state itself since the transformer parameters are available and can be kept as a template. In practice when actual partial discharge takes place in the transformer in operation the voltage can be recorded in the input terminal of transformer using a CRO and can be compared with the template voltages to find out where partial discharge is occurred and also whether it is a inter turn discharge or it is ground to node discharge.

**SIMULATION OF INTERTURN INSULATION FAILURE:**

For the purpose of certification as per IS2026 impulse test has to be conducted on power transformers. The main components of tests are a standard 1/50 impulse voltage of prescribed magnitude followed by chopped voltage test, which is again followed by 1/50 impulse voltage test. When the chopped voltage test is applied, the transformer winding will be severely stressed. So if the transformer winding fails at
any location this will be shown in oscillogram taken. Usually for this purpose oscillogram of the neutral currents of the full impulse voltage applied before and after the chopped voltage is compared. If there is no change in the two oscillogram, the transformer is deemed to have passed the impulse voltage test. If there is a change that means it is due to the inter turn insulation failure or major insulation failure. However from the oscillogram one can not find out where the failure has occurred. Digitally simulating the various inter turn insulation failures by shorting the corresponding two nodes current records can be obtained and this can be compared with the oscillogram obtained in the actual impulse voltage test which will help to find out the actual location of the inter turn insulation failure.

We have simulated the partial discharge across every node and node, node and ground in transformer winding. So that in practice if we found any partial discharge in transformer windings, we can compare it with our templated results and faults can be easily diagnosed.

CONCLUSION:

This paper finds an appropriate method to find the stress in transformer winding and the digital simulation of various faults are obtained.

APPENDIX:

RESULTS:
Waveform without inter turn insulation failure
Inter turn insulation failure between first and second node

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